Influence of Soil Electrogeological Properties on the Impact Point of Lightning Discharge

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Abstract—In this paper, we present some results of investigations carried out on the effect of soil heterogeneity and conductivity on the strike point of lightning discharge using mathematical and experimental model. The mathematical model simulates the propagation of negative downward leader, it shows that the electric field in the soil interface depends on the conductivities ratio of the heterogeneous soil. In the experimental model we observed that the conductivities of the heterogeneous soil and the distance from the interface, affect the discharge attraction to the soil interface. The use of experimental model parameters in the mathematical model, shows a concordance between the two models.

Index Terms—Earth conductivity, Earth heterogeneity, Electric field intensity, Lightning impact, Stepped leader modeling.

1 INTRODUCTION

In lightning protection field, many researches interested on experimental modelling of lightning discharge phenomenon [1]–[3]. These studies had shown that the electrogeological properties of the soil, affect the breakdown voltage of a rod-plane air gap arrangement. The more studied case is the negative descending strike. The stepped leader was represented by a rod under negative lightning impulse voltage. The previous observations led to the study of the capture zone of horizontal and vertical earthed rod [4], [5]. Both heterogeneous and discontinuous earths were studied. It has been found that the capture zones of a lightning conductor could be, greater than defined by the classical electrogeometrical model. Also, electric field measurements using a capacity probe, had shown that the electric field intensity depends largely on the earth nature, heterogeneous or homogeneous, and on its conductivity [6].

In this paper, we present two models for the lighting discharge phenomenon, mathematical and experimental one. The mathematical model simulates the propagation of a negative downward stepped leader from the cloud toward the soil in real time, while the experimental model simulates the final jump of the downward stepped leader. Thereafter, we present a comparison between the two models.

2 LIGHTNING DISCHARGE PHENOMENON

The formation of storm cloud is due to the instability of the atmosphere [7]. These clouds develop when warm moist air near the surface of the earth rises and replaces the dense air above.

In good weather, electric field intensity on the ground surface is of the order of 0.13kV/m, in the presence of a storm cloud, this value can reach a value of 1.5 to 2kV/m [8]. Atmospheric discharges can take place inside the cloud or between the cloud and the earth.

The discharges to earth, can be separated into two parts:

- The pre-discharge, which establishes the link between the cloud and the ground.
- The main discharge, that neutralizes the charges of the cloud or part of them, following the pre-discharge path.

These discharges can be classified into four main types [9], according to the polarity of the discharged cloud and the direction - downward or upward – of the leader.

Downward leader may occur when the electric field is sufficiently intense at the base of the cloud to trigger an electrical discharge toward earth. This discharge is spread by jumps which take place in a time of about $10^{-6}s$, ranging in length from 10m to 200m [10], [11]. Each jump is oriented towards the zone where the electric field is the most intense in front of the leader tip [12]. Between two successive jumps there is a break of about $10^{-5}s$ to $10^{-4}s$. The average speed of the stepped leader is about 10 to 130cm/μs [10].

Most often, before the descending stepped leader establishes a connection between the cloud and earth, there is an upward leader, which starts towards the descending stepped leader from the point on the ground where electric field intensity reaches the critical value. The point in space where the stepped leader tip, is called the “critical point” at this moment. Thus, the last jump of the stepped leader is executed from this critical point [13].

As soon as this connection takes place, the ionized channel is established and the charges of opposite polarity neutralize the charges of the descending leader. This characterizes the main discharge, which propagates continuously at the order of 1m/μs [10].

3 THE ELECTROGEOLOGICAL PROPERTIES OF THE SOIL

3.1 Soil structure

Most often the soil structure is complex composed of several materials, each of these materials is characterized by [14]:

- A relative permeability $\mu_r$ : varies in a small range, with a maximum value of 5.
A relative permittivity $\epsilon_r$ : varies from unity to the hundred.
A conductivity $\delta$ : varies in a wide range, from $10^{-16}$ to $10^6$ s/m.

3.2 Continuity equation and time constant
The application of an electric field to a material causes a masses displacement of the charge carriers. Conservation of the masses, means conservation of charges, which is expressed mathematically by the continuity equation (equation (1)) [15]. The differential form of this equation, which can be obtained from Maxwell’s equations, is as follows:

$$\frac{\partial Q}{\partial t} + \frac{1}{\epsilon/\delta} Q = 0$$

Or the report $T_c = \epsilon/\delta$ represents a time constant. By solving this equation, we found for the charge density:

$$Q = Q_0 \cdot e^{-(t-t_0)\delta/\epsilon}$$

Where:
- $\epsilon$ the electrical permittivity.
- $\delta$ the electrical conductivity.
- $t_0$ the initial time.
- $Q_0$ the initial charge density.

It is noted that the time constant increases with the permittivity and decreases with the increase of the conductivity. The conduction in liquid and solid materials is the result of charge carriers movement. The movement of these charges creates a current density in the material.

$$\vec{j} = \delta \cdot \vec{E} + \frac{\partial \vec{D}}{\partial t}$$

Where:
- $\vec{j}$ the current density.
- $\vec{E}$ the electric field.
- $\partial \vec{D}/\partial t$ the displacement current density.

The displacement current is not taken into consideration in the work.

By introducing the extreme values of conductivity and permittivity into the expression of the time constant, we obtain its minimum and maximum magnitudes:

$$\begin{cases} T_{c_{\text{min}}} = 1.4 \cdot 10^{-19} \text{s} \\ T_{c_{\text{max}}} = 9.3 \cdot 10^6 \text{s} \end{cases}$$

Comparing these values with the characteristic times of lightning discharge propagation, we find that the displacement of the charges, in the materials having a time constant smaller or equal to the time necessary for a jump ($10^{-6}$s), must be done at the time of the jump. While for materials with a large time constant, the charges move very slowly compared to the evolution of the stepped leader.

4 Mathematical model
The mathematical model was established to study the influence of soil heterogeneity on the electric field intensity on the soil during the advancement of the descending leader. The critical value of this field is $5 \text{kV/cm}$ for the negative descending leader and $3 \text{kV/cm}$ for the positive descending leader [13].

4.1 Mathematical description of the phenomenon
The system is represented by a planar electrode capacitor. The descending leader is represented by a vertical rod.

The link between the various physical factors of the system is represented by the block diagram of Fig.1.

During the waiting time between two jumps of the stepped leader, the electric field intensity varies with the variation of the charge density according to the block diagram. The distribution of charges density obtained before the next jump gives the initial charge density for the new position of the descending leader.

4.2 The studied domain
The domain is chosen in such a way that it has a cylindrical shape whose axis is the lightning descending leader, limited at the top by the plane of the clouds having the potential $V_{\text{cloud}}$ and at the bottom by the reference plane having the potential zero, with the following simplifications:
- The descending leader spreads vertically, and its potential is identical to that of the cloud.
- The heterogeneous soil is composed of two different materials and their interface is a cylindrical surface whose axis is confused with that of the descending leader.

As the cylindrical symmetry is imposed on the area, the resolution of the system is a two-dimensional. The boundary conditions are represented in Fig.2.

The conductivities values of the soil components studied were limited by the time constant constraint of the continuity equation. This constant had to be for each component
larger than the time necessary for a jump of the descending leader \((10^{-6}s)\). This condition with the choice of the dielectric constant equal to unity, has given us the maximum conductivity, which we can choose:

\[
\delta_{\text{max}} < 8.85 \times 10^{-6} \frac{S}{m}
\]

(5)

This choice is necessary so that the propagation of the descending leader can be considered as the superposition of static states.

The values of conductivities examined, the time constants and the time steps are noted in Table 1. The height \(h\) varied from 400m or 300m to 150m for each configuration with a waiting time between two heights about \(10^{-4}s\) [7], [11]. For each set of conductivities, the distance of the interface varies from 300 to 500m. The potential of the cloud was taken of 90MV.

<table>
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<tr>
<th>Case number</th>
<th>(\delta_1 \left(\frac{\Omega}{m}\right))</th>
<th>(TC_1) (\times 8.85(s))</th>
<th>(\delta_2 \left(\frac{\Omega}{m}\right))</th>
<th>(TC_2) (\times 8.85(s))</th>
<th>(dT) (\times 10^{-6}(s))</th>
</tr>
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<td>1</td>
<td>(10^{-9})</td>
<td>(10^{-3})</td>
<td>(10^{-6})</td>
<td>(10^{-6})</td>
<td>7.5</td>
</tr>
<tr>
<td>2</td>
<td>(10^{-12})</td>
<td>1</td>
<td>(10^{-6})</td>
<td>(10^{-6})</td>
<td>7.5</td>
</tr>
<tr>
<td>3</td>
<td>(10^{-12})</td>
<td>1</td>
<td>(10^{-7})</td>
<td>(10^{-5})</td>
<td>75</td>
</tr>
</tbody>
</table>

The finite element method is used for the resolution of the problem following the flowchart in Fig.3.

Where:
- \(h\) the height of the descending leader.
- \(Q_{\text{cloud}}\) amount of charge in the cloud.
- \(E\) the intensity of the electric field on the soil surface.
- \(T_a\) the waiting time between two successive jumps of the descending leader.
- \(T_i\) the initial time corresponding to a position of the descending leader.

### 4.3 Results

Fig.4 represents the electric field intensity at the interface according to the position \(x\). We notice a slight increase in the electric field intensity when the position \(x\) decreases. This applies in the three cases reviewed.

Fig.5 shows the variation of the electric field intensity at the interface for \(x = 500\) m. Note that the electric field varies according to an exponential function.

### 5 EXPERIMENTAL MODEL

#### 5.1 Model presentation

The experimental model, representing the final jump of the downward leader on the heterogeneous soil, consists of a wooden box whose dimensions are \(1\text{m} \times 1.5\text{m} \times 0.15\text{m}\), and a metal rod with a diameter of 6 mm, whose end was a hemisphere of the same diameter.

This box was filled with two different materials having a well-defined interface, which made it possible to simulate the flat heterogeneous soil (Fig.6). The two materials were galvanized steel and dry or wet sand.

Before the descending leader reaches the critical point, it evolves independently of the soil structure. This makes it possible to simulate it by a vertical rod whose tip corresponds to the critical point. It is from this point that the final jump is made. This forced us to apply pulses with a sufficiently large peak value \((U_{100\%})\).

#### 5.2 Tests

The tests were carried out in the H.V laboratory of National Polytechnic School of Algiers. We used a shock generator \((-1.2/50\mu s)\) with a nominal voltage of 600kV.

![Fig. 3. Flowchart of the digital program.](image)

![Fig. 4. Electric field intensity at the interface according to the position \(x\) of the interface, in a time equal to 390\mu s after the beginning of the phenomenon \(E_0 = 5\text{kV/cm}\).](image)
Fig. 5. Variation of the electric field intensity at the interface in time for \( x = 500 \text{m} \), \( E_0 = 5 \text{kV/cm} \). The arrows indicate the instants of the jumps.

The position of the rod is defined by the height \( h \) and the distance \( x \), with a sand thickness of 15cm.

During our tests we observed three types of disruptive discharge:
1. Direct discharge to the sand or the interface.
2. Direct discharge to the sand then slippery to the interface.
3. Discharge to interface and sand at a time.

We also observed that the number of discharges that go to the interface decreases as the distance \( x \) increases.

For each system configuration we determined the frequency \( f_i \) of impact at the interface. This calculation was done using the following expression:

\[
 f_i = \frac{n}{50} \tag{6}
\]

Where \( n \) is the number of discharges that reached the interface.

5.3 Results

By taking the frequencies \( f_i \) according to the report \( x/h \) in a Gauesso-arithmetic scale, for the dry and wet sand, we obtained straight lines (Fig.7).

These results suggest that the distribution of impacts at the interface follows the normal distribution. The regression slope is greater for wet sand than for dry sand.

These results show that the influence of the interface on the attraction of lightning discharges increases with the increase of the soil heterogeneity degree.

6 USE OF THE EXPERIMENTAL MODEL CHARACTERISTICS FOR THE MATHEMATICAL MODEL

6.1 Study purpose

The main goal of this study is to determine, using the mathematical model, the variation of the electric field intensity at the heterogeneous soil surface for the experimental model configurations.

As the pulses applied to the system were at \( U_{100\%} \) level, the disruption had to happen on their front or at the edge at their peak. Thus, the time between the application of a
pulse and the disruption of the system could only be 1.2µs maximum.

This fact leads us to apply a step to the mathematical model system and to determine the electric field intensity on the heterogeneous soil surface 1.2µs after the application of this step.

6.2 Comparison between the two models
Comparing the experimental model to the mathematical model, we can see that there are two important differences between the two models:

- The heterogeneous soil structure: In the case of the mathematical model the structure is characterized by concentric cylindrical layers, which is not the case for the experimental model.
- The boundary conditions: For the experimental model, storm cloud was not taken into account.

Also, galvanized steel has a very low time constant compared to the length of the step, which leads us to set the potential of the steel surface equal to zero. For the sand conductivities, we use the mean values of the conductivities measured in the experimental model. For the dielectric constant, we have chosen the dielectric constant of dry sand measured in the experimental model. For the dielectric conductivities, we use the mean values of the conductivities as the stepped leader evolves.

We note from Fig. 9, a decrease in the report $E_i/E_p$ when $x$ increases. This is in agreement with the experimental results. Indeed, when the axis of the descending leader moves away, the probability of the discharge decreases.

6.3 Results
The electric fields $E_p$ increases as $x$ increases (Fig.8). This is due to the increase in the amount of charge contained in the sand. For the two heights studied, the electric field $E_p$ is higher when the sand is wet. This is due to the ease of charge carriers movement when the conductivity of the sand increases.

We note from Fig. 9, a decrease in the report $E_i/E_p$ when $x$ increases. This is in agreement with the experimental results. Indeed, when the axis of the descending leader moves away, the probability of the discharge decreases.

7 Conclusion
In this work, we have established two models representing the phenomenon of lightning discharge, mathematical and experimental model.

The mathematical model allows us to calculate the electric field intensity on the heterogeneous soil surface during the evolution of the lightning leader. For all the configurations studied, we obtained:

- An exponential increase in field strength at the interface as the stepped leader evolves.
- Electric field intensity at the interface decreases as the distance $x$ increases.

Using the experimental model, we performed a statistical study concerning the frequency of negative lightning discharges at the heterogeneous soil interface. From these tests we received a distribution at the interface of lightning discharges which follows a normal distribution according to the ratio $x/h$.

Using the electrical characteristics of the experimental soil components in the mathematical model, we were able to find a correlation between the two models.

References


